THERMAL LAYERS NEAR A FRICTION SURFACE OF BODIES IN HIGH-VELOCITY SLIP

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Thermal layers generated when one body slips over the surface of another are considered. Exactness of the solutions obtained by the methods of the boundary layer theory is analyzed.

In designing machines, it is important to know the temperature fields emerging in friction units, especially with high velocities of slip. In a moving body the thermal energy is concentrated near the contact surface, inside a narrow layer expanding in time (fairly short time intervals are concerned). Inside a counterbody, in the case of a specified relative velocity, a thermal layer expanding from the front face of the moving body is generated after a short period of time, during which the process sets up (see Fig. 1). A considerable number of studies [1-5] deal with the determination of relevant temperature fields. The following mathematical model is used more often than others. The body, contacting the counterface, instantaneously passes from an initially immobile state to a state of motion at an assigned velocity. The surface heat flux density, assumed to be a constant value, is generally imposed on the contact surface, which is a heat source. In identifying the thickness of thermal layers a difficulty is encountered related to the fact that the variable boundary of the thermal layer is a conventional quantity, i.e., it is an isotherm on which the temperature value is ten or, for example, a hundred times smaller than the value registered on the boundary with an imposed heat flux or temperature. A certain arbitrariness in selecting the boundary is inherent in boundary layer problems altogether. Since the rate of heat propagation is considered infinite, on the origination of a point or plate source at the time $t = 0$ the temperature variation, strictly speaking, is other than zero for any finite values of the coordinate and nonzero values of t. However, the bulk of the thermal energy is in the zone whose width δ, as the dimensional analysis [6] reveals, is proportional to the quantity $\sqrt{at}$ with one or another coefficient

$$\delta = k\sqrt{at}.$$

Zel'dovich and Raizer [6] indicated the coefficient $k = 2$ for an instantaneous point source. For a linear heat flux, Fazekas [1] obtained $k = 1.75$ by a graphoanalytic method. Il'yushin and Ogibalov [2] determined the layer width δ by expanding the expression for temperature in an integral form in terms of the coordinate and restricting themselves to a linear approximation, and it appeared here that $k = 2/\sqrt{\pi}$. Chichinadze [1], when solving the problem by the Fourier method, also confined himself to initial terms of the expansion and derived $k = 1.73$.

Drozdov [3] analyzed the dependence of the thermal layer depth on how it is agreed to draw its boundary. If the temperature at the depth δ is taken to be 10% of that on the friction surface, then $k = 1.94$, and for the temperature falling to 1%, $k = 3.2$.

Balakin [4] attempted to calculate the portion of the entire thermal energy concentrated in the layer of thickness δ. As a consequence of the error in calculations, the quantity at issue turned out to depend on two parameters, k and the Fourier number, whereas in reality it must be dependent only on k.

The current study has addressed the problem of a thermal layer boundary in a different way. The temperature field in the thermal layer may be described approximately by a polynomial of some degree. The higher the polynomial degree and the more precise is the definition, the farther the layer boundary passes into a low-temperature region, i.e., the layer per se is thicker. It is of interest to clarify how the conventional thickness of the thermal layer depends on the degree of the polynomial, approximating the temperature variation inside it.

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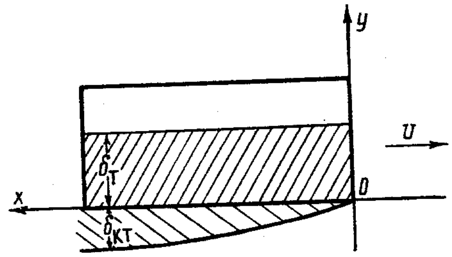

Fig. 1. Diagram of thermal layers originating in friction, in a body (the layer thickness is δ_b) and in the counterbody (δ_{cb}).

When one body slips at a high velocity over the surface of another, a surface melting may occur. We consider only two limiting cases: 1) the body slips over the surface of a low-melting material, and a liquid film spreads over the entire contact surface; and 2) there is no melting. In the first case, the temperature of the contact surface, approaching the melting point of the counterbody material, remains unchangeable, and in the second case, the friction surface is a source of the heat flux, whose density we will assume to be a constant value.

A mathematical model corresponding to the first case is formulated as follows. We write the heat conduction equation

$$\frac{\partial T}{\partial t} - a\frac{\partial^2 T}{\partial y^2} = 0, \tag{1}$$

the initial condition:

$$\text{for } t = 0 \quad T = 0, \tag{2}$$

and the boundary conditions:

$$\text{for } t > 0 \quad T|_{y=0} = T_0, \quad T|_{y=\infty} = 0. \tag{3}$$

The problem solution is known (see, for example, [7], where the same problem arises in computing a liquid velocity field near the wall, which suddenly began to move):

$$\frac{T}{T_0} = \frac{2}{\sqrt{\pi}}\int_\eta^\infty \exp(-\xi^2)\,d\xi = \operatorname{erfc}\eta, \quad \eta = \frac{y}{2\sqrt{at}}. \tag{4}$$

The function erfc η, referred to as an additional probability integral, is tabulated (see [8]).

Hence it is not difficult to see time variations in the heat flux from the contact surface to the body under the conditions of constant temperature at the boundary:

$$q_0 = -\lambda\left.\frac{\partial T}{\partial y}\right|_{y=0} = \frac{\lambda T_0}{\sqrt{\pi a t}} = T_0\sqrt{\frac{\lambda\rho c}{\pi t}}. \tag{5}$$

The amount of thermal energy absorbed by the body in a band of width δ per unit area of the surface is

$$Q_{ac\delta} = \rho c\int_0^\delta T\,dy = \frac{2}{\sqrt{\pi}}T_0\sqrt{\lambda\rho c t}\left(1 - \sqrt{\pi}\,\operatorname{ierfc}\frac{\delta}{2\sqrt{at}}\right). \tag{6}$$

A definition and tables of the functions ierfc x, i^2erfc x, etc. may be found in [8, 9]. The multiplier in front of the brackets is the total amount of energy, which entered the body through unit area of the contact surface Q_{ac}. Hence, a portion of the entire thermal energy, contained inside the layer of thickness δ, is equal to

$$\frac{Q_{ac}\,\delta}{Q_{ac}} = 1 - \sqrt{\pi}\,\mathrm{ierfc}\,\frac{\delta}{2\sqrt{at}}\,. \tag{7}$$

Let us introduce the quantity δ_1 by definition

$$\delta_1 = \int_0^\infty \frac{T}{T_0}\,dy \tag{8}$$

and call it the displacement thickness for the thermal layer. It is assumed that the temperature is reckoned from the level of the surrounding medium. The quantity δ_1 is an analog of the displacement thickness in hydrodynamics [7]

$$U\delta_1 = \int_0^\infty (U - v)\,dy,$$

which characterizes the distance over which streamlines of the external flow (U is the velocity of the external flow) shift from the body in consequence of the boundary layer generation.

Substituting the expression for temperature (4) into Eq. (8) yields

$$\delta_1 = 2\sqrt{at}\,\mathrm{ierfc}\,0 = \frac{2}{\sqrt{\pi}}\sqrt{at}. \tag{9}$$

The dependence of the temperature T on the coordinate y in the thermal layer can be approximated by a polynomial in y. By way of example, we consider the case with the second-degree polynomial

$$T' = a_0(t) + a_1(t)\,y + a_2(t)\,y^2.$$

Using three boundary conditions

$$T'\,|_{y=0} = T_0;\ T'|_{y=\delta} = 0;\ \left.\frac{\partial T'}{\partial y}\right|_{y=\delta} = 0,$$

we define three coefficients a_0, a_1, and a_2, and arrive at

$$T' = T_0\left(1 - 2\frac{y}{\delta} + \frac{y^2}{\delta^2}\right) = T_0\left(1 - \frac{y}{\delta}\right)^2. \tag{10}$$

Taking into account equality (8), it is natural to require that any approximation of the dependence of the temperature on the coordinate in a layer of width $\delta T'(y)$ satisfy the relationship

$$\delta_1 = \int_0^\delta \frac{T'}{T_0}\,dy. \tag{11}$$

Substituting Eq. (10) into Eq. (11) results in

$$\delta_1 = \frac{1}{3}\,\delta.$$

TABLE 1. Displacement Thickness for Thermal Layer δ_1 with Constant Temperature or Heat Flux Density at the Boundary

Condition at the boundary	Body	Counterbody
$T_0 = \text{const}$	$\frac{2}{\sqrt{\pi}}\sqrt{at}$	$\frac{2}{\sqrt{\pi}}\sqrt{\frac{ax}{U}}$
$q_0 = \text{const}$	$\frac{\sqrt{\pi}}{2}\sqrt{at}$	$\frac{\sqrt{\pi}}{2}\sqrt{\frac{ax}{U}}$

TABLE 2. Approximation of Temperature Fields in Thermal Layers by Polynomials of the Form $T_0(1 - y/\delta)^n$

Polynomial degree n	Thermal layer width, δ	$h=\frac{\delta}{\sqrt{at}}$		$T\|_{y=\delta}/T_0$		$1-Q_{ac\delta}/Q_{ac}$	
		T_0=const	q_0=const	T_0=const	q_0=const	T_0=const	q_0=const
0	δ_1	1,13	0,89	0,425	0,406	0,30	0,33
1	$2\delta_1$	2,26	1,77	0,111	0,126	0,058	0,084
2	$3\delta_1$	3,38	2,66	0,016	0,029	0,007	0,016
3	$4\delta_1$	4,15	3,54	0,0014	0,0051	0,001	0,002
4	$5\delta_1$	5,64	4,43	0,00007	0,0006	0,0002	0,0003

Let us find the error arising when we take $T = 0$ at $y = \delta = 3\delta_1$. In accordance with Eq. (4), the actual value of the temperature in dimensionless form is

$$T|_{y=\delta}/T_0 = \operatorname{erfc}\frac{3}{\sqrt{\pi}} = 0.016.$$

Similarly, the portion of the thermal energy contained in a layer of thickness δ for the case $\delta = 3\delta_1$, under the condition of constant temperature at the contact boundary, equals

$$Q_{ac\delta}/Q_{ac} = 1 - \sqrt{\pi}\,\operatorname{ierfc}\frac{3}{\sqrt{\pi}} = 0.993.$$

It is easily seen that, in the general case, the approximating polynomial of degree n having the form

$$T' = T_0\left(1 - \frac{y}{\delta}\right)^n \tag{12}$$

satisfies the boundary conditions

$$T'|_{y=0} = T_0;\ T'|_{y=\delta} = 0;\ \left.\frac{\partial T'}{\partial y}\right|_{y=\delta} = 0;\ \ldots;\ \frac{\partial^{n-1}T'}{\partial y^{n-1}} = 0, \tag{13}$$

and from relationship (11) follows

$$\delta = (n + 1)\,\delta_1. \tag{14}$$

Up to this point we considered the problem under the condition of constant temperature at the contact surface, when a moving body is in contact with a liquid film. Now we examine the case of dry friction. Let the velocity of relative movement be invariable. From the outset of the movement, a heat flux into the body arises, whose density q_0 is assumed constant along the body length and in time. The mathematical problem is described by Eq. (1) with the initial condition (2) and boundary conditions

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = -\frac{q_0}{\lambda}; \quad \left.\frac{\partial T}{\partial y}\right|_{y=\infty} = 0. \tag{15}$$

By differentiating both sides of Eq. (1) with respect to y, we arrive at an equation for the flux density q(y, t). Here, the boundary conditions (15) for q turn out to be conditions of the first kind. The solution has the form

$$q = -\lambda \frac{\partial T}{\partial y} = q_0 \,\mathrm{erfc} \frac{y}{2\sqrt{at}}. \tag{16}$$

By subsequently integrating both sides of the last of equalities (16) with respect to y from y to infinity, we obtain an expression for temperature (see, for example, [3] or [10, 11])

$$T = \frac{2q_0}{\lambda} \sqrt{at}\, \mathrm{ierfc} \left(\frac{y}{2\sqrt{at}}\right). \tag{17}$$

At the boundary y = 0

$$T_0 = \frac{2}{\sqrt{\pi}} q_0 \sqrt{\frac{t}{\lambda\rho c}}. \tag{18}$$

The amount of thermal energy concentrated inside a band of width δ per unit area of the surface is equal to

$$Q_{ac\delta} = q_0 t \left(1 - 4\, \mathrm{i^2erfc} \frac{\delta}{2\sqrt{at}}\right). \tag{19}$$

The displacement thickness for the thermal layer with q_0 = const differs somewhat from the corresponding thickness in the case T_0 = const:

$$\delta_1 = 2\sqrt{at}\, \mathrm{i^2erfc}\, 0/\mathrm{ierfc}\, 0 = \frac{\sqrt{\pi}}{2}\sqrt{at}. \tag{20}$$

Further on, we examine the temperature field in the counterbody region under the liquid layer (T_0 = const) or, in the case of dry friction (q_0 = const), under the contact surface. The process is assumed established and quasisteady. The temperature field in the two-dimensional case in a coordinate system moving together with the body at the velocity U is defined by the equation

$$U\frac{\partial T}{\partial x} = a\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2}\right).$$

Let us confine ourselves to the cases when the Peclet number with a characteristic length in the direction of x is large

$$\mathrm{Pe} = Ul/a \gg 1.$$

Here, the heat flux, associated with the heat conduction, in the x direction is much smaller than in the y direction. Neglecting a small term in the original equation, we obtain a simpler equation

$$\frac{\partial T}{\partial x} = \frac{a}{U}\frac{\partial^2 T}{\partial y^2}. \tag{21}$$

Boundary conditions for the liquid film have the form

$$\begin{aligned} &\text{when } x<0 \; T=0; \\ &\text{when } x \geqslant 0 \; T|_{y=0}=T_0; \; T|_{y=\infty}=0. \end{aligned} \tag{22}$$

It is not difficult to see that, mathematically, this problem is equivalent to a nonstationary problem on heat propagation in a body with a designated temperature at the boundary, i.e., to Eq. (1) with boundary conditions (2) and (3). Only the replacement $t \rightarrow x$ and $a \rightarrow a/U$ should be made. Therefore, we may immediately write the solution analogous to Eq. (4)

$$\frac{T}{T_0} = \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{U}{ax}}\right). \tag{23}$$

For the heat flux at the boundary $y = 0$ with a specified temperature we have

$$q_0 = T_0 \sqrt{\frac{\lambda \rho c U}{\pi x}}. \tag{24}$$

Here, within the counterbody under the moving body (more precisely, under the melt boundary), in a layer of thickness $\delta(x)$, energy is concentrated which, when referred to unit surface area, equals

$$Q_{ac\delta} = \frac{2}{\sqrt{\pi}} T_0 \sqrt{\frac{\lambda \rho c x}{U}} \left[1 - \sqrt{\pi} \operatorname{ierfc}\left(\frac{\delta}{2}\sqrt{\frac{U}{ax}}\right)\right]. \tag{25}$$

The displacement thickness for the thermal layer in the counterbody at T_0 = const is

$$\delta_1 = \frac{2}{\sqrt{\pi}} \sqrt{\frac{ax}{U}}. \tag{26}$$

In the dry friction case with q_0 = const, based on Eqs. (17)-(20), we correspondingly derive for the counterbody

$$T = \frac{2q_0}{\lambda} \sqrt{\frac{ax}{U}} \operatorname{ierfc}\left(\frac{y}{2}\sqrt{\frac{U}{ax}}\right); \tag{27}$$

$$T_0 = \frac{2}{\sqrt{\pi}} q_0 \sqrt{\frac{x}{\lambda \rho c U}}; \tag{28}$$

$$Q_{ac\delta} = q_0 \frac{x}{U}\left[1 - 4i^2\operatorname{erfc}\left(\frac{\delta}{2}\sqrt{\frac{U}{ax}}\right)\right]; \tag{29}$$

$$\delta_1 = \frac{\sqrt{\pi}}{2} \sqrt{\frac{ax}{U}}. \tag{30}$$

The expression for the temperature T_0 at the contact surface (28) coincides with that obtained by Eger [5] in a different way.

Clearly, in approximating the temperature–coordinate relation inside the thermal layer by a polynomial of the type $T_0(1 - y/\delta)^n$, the relative error both as to the temperature at the boundary $y = \delta$ and as to the thermal energy portion, concentrated inside the layer, does not depend on whether the thermal layer in the body or in the counterbody is considered.

The magnitude of the error, however, is dependent to some extent on the conditions at the contact surface, for example, on whether the temperature or the heat flux density at the boundary remains unchanged. In the case with T_0 = const

$$T|_{y=\delta}/T_0 = \operatorname{erfc} \frac{n+1}{\sqrt{\pi}} ; \tag{31}$$

$$1 - \frac{Q_{ac\delta}}{Q_{ac}} = \sqrt{\pi} \operatorname{ierfc} \frac{n+1}{\sqrt{\pi}} . \tag{32}$$

For q_0 = const

$$T|_{y=\delta}/T_0 = \sqrt{\pi} \operatorname{ierfc} \frac{n+1}{\sqrt{\pi}} ; \tag{33}$$

$$1 - \frac{Q_{ac\delta}}{Q_{ac}} = 4 i^2 \operatorname{erfc} \frac{n+1}{\sqrt{\pi}} . \tag{34}$$

Table 1 gives displacement thicknesses for thermal layers in the body or counterbody as applied to the considered cases. Table 2 summarizes numerical results, characterizing the approximation accuracy for various degrees of the approximating polynomial up to n = 4. The thermal layer width with a constant heat flux at the boundary appears to be smaller than in the case of a designated temperature, in the ratio of $\pi/4$. A linear approximation of the thermal layer temperature produces an error as to the concentrated energy equal to 6-8%, and a quadratic approximation increases the layer thickness and reduces the error to 0.7-1.6%.

Thus, it is established that the conventional width of the thermal layer correlates in a multiple way with the degree of the polynomial approximating the temperature–coordinate relationship, and accuracy of the relevant approximation is estimated.

NOTATION

t, time; x, y, coordinates; l, length of a slipping body (along x); δ, conventional thickness of the thermal layer; T, temperature reckoned from the level of the surrounding medium; q, heat flux density, W/m^2; T_0, q_0, temperature and heat flux density near the friction surface; U, velocity of the relative movement of bodies, m/sec; ρ, substance density, kg/m^3; c, specific heat, $J/(m^3 \cdot K)$; λ, thermal conductivity, $W/(m \cdot K)$; $a = \lambda/(\rho c)$, thermal diffusivity, m^2/sec.

REFERENCES

1. A. V. Chichinadze, Calculation and Study of External Friction in Deceleration [in Russian], Moscow (1967).
2. A. A. Il'yushin and P. M. Ogibalov, Plastoelastic Deformations of Hollow Cylinders [in Russian], Moscow (1960).
3. Yu. N. Drozdov, Mashinovedenie, No. 2, 71-79 (1972).
4. V. A. Balakin, Friction and Wear at High Velocities of Slip [in Russian], Moscow (1980).
5. J. C. Jaeger, Proc. R. Soc. New South Wales, **76**, Pt. III, 203-224 (1942).
6 Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and of High-Temperature Hydrodynamic Phenomena [in Russian], Moscow (1966).
7. G. Schlichting, Boundary-Layer Theory, 6th edn., McGraw-Hill, New York (1968). (1974).
8. H. Carslaw and J. Jaeger, Conduction of Heat in Solids, 2nd edn. Oxford University Press (1959).
9. A. V. Lykov, Heat Conduction Theory [in Russian], Moscow (1967).
10. V. A. Balakin, Basics of Strength of a Surface Layer (Friction and Wear) [in Russian], Gomel' (1974).
11. A. V. Chichinadze and V. M. Goryunov, Thermal Dynamics of Friction [in Russian], Moscow (1970).